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**Vibration and Control of Axially Moving Belt System. 3rd Report, Analysis by Parametric Excitation.**

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# 走行ベルト系の振動と制御*
## (第3報，係数励振による振動解析)

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# Vibration and Control of Axially Moving Belt System
## (3rd Report, Analysis by Parametric Excitation)

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High speed operation is required for industrial machines equipped with belt driving systems. The belt is, however, subject to a speed limitation barrier due to the occurrence of bending mode vibration resonances caused by machinery error, e.g., pulley eccentricity. To combat these resonances of the linear system, the parametric excitation method providing the belt tension fluctuation is proposed through the concept of the open loop vibration control theory. This analysis considers two types of fluctuation excitation featured by the frequency synchronized with one or two times the pulley revolution. The former is the most effective whereas the latter imparactical. The analytical results which are verified by well tuned experiments look promising.

***Key Words***: Vibration of Moving Body, Vibration Control, Parametric Excitation, Moving Belt

## 1. は じ め に

ゴムベルトを動力伝達媒体とするベルト駆動装置は，種々の産業機器に広く用いられている．特に走行ベルト系は，目的，用途に応じてさまざまな形態をとり，また要求される性能も多用化している．走行ベルトは高精密化，高速化が図られ，従来問題とされていなかったベルトの横振動の共振が大きな問題となって来ている．振動抑制制御に関する研究として，Chungらが境界点変位入力によるアクティブ振動制御の理論解析を行っている(1)．また，Rahnらは係数励振による振動制御に関する理論解析を行っている(2)．これらは状態フィードバック制御を基調としており，センサの設置や張力変動を与えるアクチュエータの形式など，実用面での問題も多いと考えられる．

本研究では，ゴムベルトなどの柔軟な媒体を対象とし，横振動を抑制することを目的とする．前報では，基礎実験と解析による比較検討を行っている(3)~(5)．本論文では，係数励振制御による振動抑制に関して1自由度での理論解析を行い，考え方の基本を明確にする．ベルト駆動用モータトルクに変動を与え，ベルトの張力変動を引き起こし，それによって振動を抑える方法で，図1に示すように実用化しやすい方法を念頭に置いている．

振動を制御する一般的な方法としては，減衰力により振動エネルギーを吸収して振動を制御する方法，発生した振動に対して，逆位相の波動を強制的に発生させて，相殺する方法などがある．前者の振動エネルギーの吸収による手法は，振動の発生をフィードバック制御する手法である．後者の波動相殺による振動制御手法は，発生した振動を打ち消すように加振するオープンループ制御をする手法である．よって両者では，その目的，用途が異なり，本論文では，一自由度系の柔軟モデルに対して，係数励振を用いてオープンルー

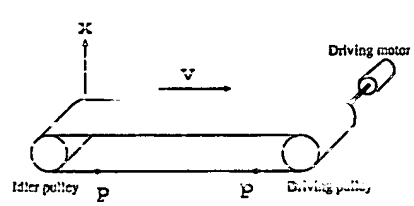


Fig. 1 Driving Belt model

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プの振動抑制のための解析を行う．

## 2. システムモデル

第1報[4]で報告したように，本研究で扱う走行ベルトは速度による影響（コリオリ力）が微小で固有振動数が速度の増加によって大きく変化しない．そのため，速度項の影響がないものとして考える．また，本ベルトシステムの加振源はアイドラプーリの偏心である．このときの運動方程式は前報により次のように書ける[5]．

$$\ddot{x}+\omega_n^2\{1+h\cos(\nu t+\alpha)\}x+2\zeta\omega_n\dot{x}$$
$$=e^*\omega^2\cos\omega t+g^* \quad \cdots\cdots\cdots(1)$$

ここで，$x$ は変位，$\omega$ は加振周波数，$\omega_n$ は共振周波数，$h$ は変調度，$\nu$ は励振周波数，$\alpha$ は位相，$t$ は時間，$e^*$，$g^*$ はそれぞれ $e^*=(2/\pi)e$，$g^*=(4/\pi)g$ となり $e$ は偏心，$g$ は重力加速度を表している．$\zeta$, $e^*$, $h$ は微少値，$\omega$ は $\omega_n$ に近い場合を仮定する．左辺の第2項は張力変動，第3項は減衰を表している．右辺の第1項は偏心遠心力を表している．非線形性の小さい場合のマーシュ方程式の安定性については，多くの論文が発表されている[6]~[8]．

本論文の主旨は，どのような張力変動のパラメータ $(h,\alpha)$ を与えることによって，共振振動制御が可能であるかを考えることである．非線形振動解析ではボゴリューボフ，ミトロポリスキーによる漸近展開法[9]に基づいて行う．そして，後の解析をしやすくするため，微分方程式を次のように書き換えておく．

$$\ddot{x}+\omega^2x=(\omega^2-\omega_n^2)x-2\zeta\omega_n\dot{x}+e^*\omega^2\cos\omega t$$
$$-\omega_n^2h\cos(\nu t+\alpha)x+g^* \quad \cdots\cdots\cdots(2)$$

## 3. 柔軟系の振動解析

**3・1 微分方程式の整理**　式(2)に対し，近似解として次を考える．

$$x=a\cos(\omega t+\vartheta)+\epsilon u_1(a,\vartheta,\nu t) \quad \cdots\cdots\cdots(3)$$

$a$ は振幅，$\vartheta$ は位相を表す．これらは，一定ではなく，非線形性のためゆっくり変化する変数として，以下のように仮定する．

$$\frac{da}{dt}=\epsilon A_1(a,\vartheta)+\epsilon^2A_2(a,\vartheta)$$

$$\frac{d\vartheta}{dt}=\epsilon B_1(a,\vartheta)+\epsilon^2B_2(a,\vartheta)$$

式(2)の左辺は次のように書ける．

$$\ddot{x}+\omega^2x=\epsilon\Big(-2A_1\omega\sin\phi$$
$$-2aB_1\omega\cos\phi+\frac{\partial^2u_1}{\partial t^2}+\omega^2u_1\Big)$$
$$+\epsilon^2\Big[\Big(A_1\frac{\partial A_1}{\partial a}+B_1\frac{\partial A_1}{\partial\vartheta}-aB_1^2-2a\omega B_2\Big)\cos\phi$$
$$+\Big\{-2A_1B_1-2\omega A_2-a\Big(A_1\frac{\partial B_1}{\partial a}$$
$$+B_1\frac{\partial B_1}{\partial\vartheta}\Big)\Big\}\sin\phi+2A_1\frac{\partial^2u_1}{\partial a\partial t}+2B_1\frac{\partial^2u_1}{\partial\vartheta\partial t}\Big]$$
$$\cdots\cdots\cdots(4)$$

本論文では，共振周波数と同じ周波数で加振した場合，$\nu=\omega$ と周波数の2倍で加振した場合，$\nu=2\omega$ を考える．2次の微小項を無視して，式(2)の右辺を整理すると次のようになる．なお，$\phi=\omega t+\vartheta$ とおいている．

$$(\omega^2-\omega_n^2)x=(\omega^2-\omega_n^2)a\cos\phi$$
$$-2\zeta\omega_n\dot{x}=2\zeta\omega_na\omega\sin\phi$$
$$+\epsilon\Big(A_1\cos\phi-aB_1\sin\phi+\frac{\partial u_1}{\partial t}\Big)$$
$$e^*\omega^2\cos\omega t=e^*\omega^2(\cos\vartheta\cos\phi+\sin\vartheta\sin\phi)$$

$\nu=\omega$ のとき．

$$-\omega_n^2h\cos(\nu t+\alpha)x$$
$$=-\frac{\omega_n^2ha}{2}\{\cos(2\phi+\alpha-\vartheta)$$
$$+\cos(\alpha-\vartheta)\}$$
$$-\omega^2h\epsilon u_1\cos(\omega t+\alpha)$$

$\nu=2\omega$ のとき．

$$-\omega_n^2h\cos(\nu t+\alpha)x$$
$$=-\frac{\omega_n^2ha}{2}\{\cos(\alpha-2\vartheta)\cos\phi$$
$$+\cos(\alpha-2\vartheta)\cos3\phi$$
$$-\sin(\alpha-2\vartheta)\sin3\phi$$
$$-\sin(\alpha-2\vartheta)\sin\phi\}$$

上記の式を用いて式(2)の両辺を等置していく．

**3・2 振幅と位相の解析（$\nu=2\omega$ のとき）**　励振周波数が加振周波数の2倍のとき（$\nu=2\omega$）において，$\epsilon^2$ 以上の項を無視して得られた式において，$\cos\phi$，$\sin\phi$ について比較することにより，次を得ることができる．

$$\left.\begin{aligned}\frac{da}{dt}&=-\zeta\omega_na-\frac{e^*\omega}{2}\sin\vartheta\\&\quad-\frac{\omega_n^2h}{4\omega}a\sin(\alpha-2\vartheta)\\\frac{d\vartheta}{dt}&=-\frac{\omega^2-\omega_n^2}{2\omega}-\frac{e^*\omega}{2a}\cos\vartheta\\&\quad+\frac{\omega_n^2}{4\omega}\cos(\alpha-2\vartheta)\end{aligned}\right\} \quad \cdots\cdots\cdots(5)$$

$u_1$ については，$\cos\phi$，$\sin\phi$ の基本波の係数になっていない直流と高調波成分のものから求める．

$$\epsilon\Big(\frac{\partial^2u_1}{\partial t^2}+\omega^2u_1\Big)=g^*-\frac{\omega_n^2ha}{2}\{\cos(\alpha-2\vartheta)\cos3\phi$$
$$-\sin(\alpha-2\vartheta)\sin3\phi\} \quad \cdots\cdots\cdots(6)$$

よって $\varepsilon u_1$ は上記の特解として以下のように求まる.

$$\varepsilon u_1=\frac{g^*}{\omega^2}+C_1\cos 3\psi+C_2\sin 3\psi \quad\cdots\cdots\cdots\cdots(7)$$

$$C_1=\frac{\omega_n^2ha}{16\omega^2}\cos(\alpha-2\vartheta)$$

$$C_2=-\frac{\omega_n^2ha}{16\omega^2}\sin(\alpha-2\vartheta)$$

ここで, 新変数 $u(t)=a\cos\vartheta$, $v(t)=a\sin\vartheta$ を導入する. このとき,

$$\left.\begin{aligned}&a^2=u^2+v^2\\&\vartheta=arc\cdot\tan\frac{v}{u}\end{aligned}\right\} \quad\cdots\cdots\cdots\cdots(8)$$

と書くことができる. これらを用いて式(5)を書き換える.

$$\frac{d}{dt}\begin{bmatrix}u\\v\end{bmatrix}=\begin{bmatrix}P_2 & Q_2\\R_2 & S_2\end{bmatrix}\begin{bmatrix}u\\v\end{bmatrix}+\begin{bmatrix}0\\-\dfrac{e^*\omega}{2}\end{bmatrix} \quad\cdots\cdots\cdots(9)$$

ただし,

$$P_2=-\zeta\omega_n-\frac{\omega_n^2h}{4\omega}\sin\alpha$$

$$Q_2=\frac{\omega^2-\omega_n^2}{2\omega}+\frac{\omega_n^2h}{4\omega}\cos\alpha$$

$$R_2=-\frac{\omega^2-\omega_n^2}{2\omega}+\frac{\omega_n^2h}{4\omega}\cos\alpha$$

$$S_2=-\zeta\omega_n+\frac{\omega_n^2h}{4\omega}\sin\alpha$$

**3・3 振幅と位相の解析（$\nu=\omega$ のとき）** 励振周波数が加振周波数に等しいとき（$\nu=\omega$）において, $\varepsilon^2$ 以上の項を無視して得られた式において, $\cos\psi$, $\sin\psi$ の基本波の係数について比較することにより, 次式を得る.

$$\left.\begin{aligned}\frac{da}{dt}&=-\zeta\omega_na-\frac{e^*\omega}{2}\sin\vartheta\\ \frac{d\vartheta}{dt}&=-\frac{\omega^2-\omega_n^2}{2\omega}-\frac{e^*\omega}{2a}\cos\vartheta\end{aligned}\right\} \quad\cdots\cdots\cdots\cdots(10)$$

$\varepsilon u_1$ については, 非基本波の特解として求める.

$$\varepsilon\left(\frac{\partial^2u_1}{\partial t^2}+\omega^2u_1\right)=g^*-\frac{\omega_n^2ha}{2}\{\cos(2\psi+\alpha-\vartheta)$$
$$+\cos(\alpha-\vartheta)\} \quad\cdots\cdots\cdots\cdots(11)$$

よって, 上式から $\varepsilon u_1$ は次のように書ける.

$$\varepsilon u_1=\frac{g^*}{\omega^2}-\frac{\omega_n^2ha}{2\omega^2}\cos(\alpha-\vartheta)$$
$$+\frac{\omega_n^2ha}{6\omega^2}\cos(2\psi+\alpha-\vartheta) \quad\cdots\cdots\cdots\cdots(12)$$

励振周波数が加振周波数に等しいときには, 1次の微小項までの解析では, 式(10)から見られるように変調度 $h$ の項が入らない. よって, 係数励振の効果を見い出さない. そのため, 2次までの微小項を含めた解析を行う. 式(2)の右辺第1項の2次の微小項は次のようになる.

$$\varepsilon(\omega^2-\omega_n^2)u_1=(\omega^2-\omega_n^2)\left\{\frac{g^*}{\omega^2}-\frac{\omega_n^2ha}{2\omega^2}\cos(\alpha-\vartheta)\right.$$
$$\left.+\frac{\omega_n^2ha}{6\omega^2}\cos(2\psi+\alpha-\vartheta)\right\}$$

式(2)の右辺第2項の2次の微小項は次のようになる.

$$-2\zeta\omega_n\left(\frac{da}{dt}\cos\psi-a\frac{d\vartheta}{dt}\sin\psi+\varepsilon\frac{\partial u_1}{\partial t}\right)$$

式(2)の右辺第3項の2次の微小項はない. 式(2)の右辺第4項の2次の微小項は次のようになる.

$$-\omega_n^2h\cos(\omega t+\alpha)\varepsilon u_1$$
$$=-\left(\frac{\omega_n}{\omega}\right)^2h\left\{g^*-\frac{\omega^2ha}{2}\cos(\alpha-\vartheta)\right\}$$
$$\times\cos(\alpha-\vartheta)\cos\psi$$
$$+\left(\frac{\omega_n}{\omega}\right)^2h\left\{g^*-\frac{\omega^2ha}{2}\cos(\alpha-\vartheta)\right\}$$
$$\times\sin(\alpha-\vartheta)\sin\psi$$
$$-\left(\frac{\omega_n^4}{12\omega}\right)h^2a\{\cos\psi-\cos(3\psi+2\alpha-2\vartheta)\}$$

$da/dt$, $d\vartheta/dt$ を求めると以下のようになる.

$$\left.\begin{aligned}\frac{da}{dt}&=-\zeta\omega_na-\frac{e^*\omega}{2}\sin\vartheta\\&\quad+\frac{\omega_n^4h^2a}{8\omega^3}\sin 2(\alpha-\vartheta)-\frac{\omega_n^2hg^*}{2\omega^3}\sin(\alpha-\vartheta)\\ \frac{d\vartheta}{dt}&=-\frac{\omega^2-\omega_n^2}{2\omega}-\frac{(\omega^2-\omega_n^2)^2}{8\omega^3}\\&\quad-\frac{\omega_n^4h^2}{12\omega^3}-\frac{e^*\omega}{2a}\cos\vartheta\\&\quad-\frac{\omega_n^4h^2}{8\omega^3}\cos 2(\alpha-\vartheta)+\frac{\omega_n^2hg^*}{2a\omega^3}\cos(\alpha-\vartheta)\end{aligned}\right\}$$
$$\cdots\cdots\cdots\cdots(13)$$

$u(t)$, $v(t)$ を用いて式(13)を書き換える.

$$\frac{d}{dt}\begin{bmatrix}u\\v\end{bmatrix}=\begin{bmatrix}P_1 & Q_1\\R_1 & S_1\end{bmatrix}\begin{bmatrix}u\\v\end{bmatrix}+\begin{bmatrix}W_u\\W_v\end{bmatrix} \quad\cdots\cdots\cdots\cdots(14)$$

ただし,

$$P_1=-\zeta\omega_n+\frac{\omega_n^4h^2}{8\omega^3}\sin 2\alpha$$

$$Q_1=\frac{\omega^2-\omega_n^2}{2\omega}+\frac{(\omega^2-\omega_n^2)^2}{8\omega^3}+\frac{\omega_n^4h^2}{12\omega^3}-\frac{\omega_n^4h^2}{8\omega^3}\cos 2\alpha$$

$$R_1=-\frac{\omega^2-\omega_n^2}{2\omega}-\frac{(\omega^2-\omega_n^2)^2}{8\omega^3}$$
$$\frac{\omega_n^4h^2}{12\omega^3}-\frac{\omega_n^4h^2}{8\omega^3}\cos 2\alpha$$

$$S_1=-\zeta\omega_n-\frac{\omega_n^4h^2}{8\omega^3}\sin 2\alpha$$

$$W_u=-\frac{\omega_n^2hg^*}{2\omega^3}\sin\alpha,\quad W_v=\frac{\omega_n^2hg^*}{2\omega^3}\cos\alpha-\frac{e^*\omega}{2}$$

## 4 共振時における振動解析

本論文では, 共振時において振動解析を進める. 共振時においては,

$$\omega=\omega_n$$

が成立する．以下，励振周波数 $\nu$ と加振周波数 $\omega$ に対して係数励振として，$\nu=\omega$，$\nu=2\omega$ の場合について，それぞれ定常状態の共振振動を求める．そして，その定常振動の安定性を論じる．最終的には，係数励振によって振動低減が安定に可能なものを探すことになる．

4·1　$\nu=2\omega$ のときの定常状態の振幅　共振時における励振周波数が加振周波数の2倍の場合の定常状態は，式(9)で $\omega=\omega_n$，$du/dt=0$，$dv/dt=0$ とおくことにより得られる．このときの $P_2, Q_2, R_2, S_2$ に $\omega=\omega_n$ を代入して得られるものをそれぞれ $P_{2n}, Q_{2n}, R_{2n}, S_{2n}$ とおく．$u, v$ は，次のように表すことができる．

$$\left.\begin{aligned} u&=-\frac{Q_{2n}}{P_{2n}S_{2n}-Q_{2n}R_{2n}}\frac{e^*\omega_n}{2}\\ v&=\frac{P_{2n}}{P_{2n}S_{2n}-Q_{2n}R_{2n}}\frac{e^*\omega_n}{2}\end{aligned}\right\} \cdots\cdots(15)$$

このときの振幅を $a_p$ と表すと，

$$a_p=\frac{2e}{|16\zeta^2-h^2|}\sqrt{h^2+8\zeta h\sin\alpha+16\zeta^2} \cdots\cdots(16)$$

と書ける．制御をしない場合には，$h=0$ となる．共振無制御時の振幅 $a_{p0}$ は，上式より，

$$a_{p0}=\frac{e^*}{2\zeta} \cdots\cdots(17)$$

と書ける．これは偏心に共振倍率をかけた線形系の共振振動振幅を表している．

4·2　$\nu=2\omega$ のときの振動低減と安定性　共振時においては，式(9)で $\omega=\omega_n$ として得ることができる．$h=4\zeta$ のときと $h\neq4\zeta$ の場合分けが生じる．また，$u, v$ の初期状態を $u(0)=u_0$，$v(0)=v_0$ とおく．

● $h=4\zeta$ のとき

$$\left.\begin{aligned} u(t)&=A_u+B_ut+C_ue^{-2\zeta\omega_nt}\\ v(t)&=A_v+B_vt+C_ve^{-2\zeta\omega_nt}\end{aligned}\right\} \cdots\cdots(18)$$

ただし，

$$A_u=\frac{e^*\cos\alpha}{8\zeta}+\frac{1}{2}\{u_0(1-\sin\alpha)+v_0\cos\alpha\}$$

$$B_u=-\frac{1}{4}e^*\omega_n\cos\alpha$$

$$C_u=-\frac{e^*\cos\alpha}{8\zeta}+\frac{1}{2}\{u_0(1+\sin\alpha)-v_0\cos\alpha\}$$

$$A_v=\frac{e^*}{8\zeta}(1-\sin\alpha)$$
$$+\frac{1}{2}\{u_0\cos\alpha+v_0(1+\sin\alpha)\}$$

$$B_v=-\frac{1}{4}e^*\omega_n(1+\sin\alpha)$$

$$C_v=-\frac{e^*}{8\zeta}(1-\sin\alpha)$$
$$-\frac{1}{2}\{u_0\cos\alpha-v_0(1-\sin\alpha)\}$$

振幅 $a$ が発散しないためには，$B_u=B_v=0$ でなければならない．このときの $\alpha$ は次の条件が必要になる．

$$\alpha=-\frac{\pi}{2} \cdots\cdots(19)$$

この場合には，$\alpha$ が式(19)の条件から少しでも外れると $a$ は収束しない．実際の機械では $\alpha$ の設定を全く狂わずに設定することは難しいので，$h=4\zeta$ の場合には実用的な設定であるとはいえない．

● $h\neq4\zeta$ のとき

$$\left.\begin{aligned} u(t)&=A_u+B_ue^{-\omega_n(\zeta+h/4)t}+C_ue^{-\omega_n(\zeta-h/4)t}\\ v(t)&=A_v+B_ve^{-\omega_n(\zeta+h/4)t}+C_ve^{-\omega_n(\zeta-h/4)t}\end{aligned}\right\} \cdots\cdots(20)$$

ただし，

$$A_u=\frac{2e^*h\cos\alpha}{h^2-16\zeta^2}$$

$$B_u=-\frac{e^*\cos\alpha}{h+4\zeta}+\frac{u_0}{2}(1+\sin\alpha)-\frac{v_0}{2}\cos\alpha$$

$$C_u=-\frac{e^*\cos\alpha}{h-4\zeta}+\frac{u_0}{2}(1-\sin\alpha)+\frac{v_0}{2}\cos\alpha$$

$$A_v=\frac{2e^*(4\zeta+h\sin\alpha)}{h^2-16\zeta^2}$$

$$B_v=-\frac{e^*(1-\sin\alpha)}{h+4\zeta}-\frac{u_0}{2}\cos\alpha+\frac{v_0}{2}(1-\sin\alpha)$$

$$C_v=-\frac{e^*(1+\sin\alpha)}{h-4\zeta}+\frac{u_0}{2}\cos\alpha+\frac{v_0}{2}(1+\sin\alpha)$$

振幅 $a$ が発散しないためには，式(20)の第3項が減衰するように $h<4\zeta$ でなければならない．

次に収束したときの振幅 $a$ の大きさを図2に示す．横軸には位相角 $\alpha$ をとっている．なお，$h=4\zeta$ のときの振幅は初期値を $u_0=0$，$v_0=0$ としてたときのもの

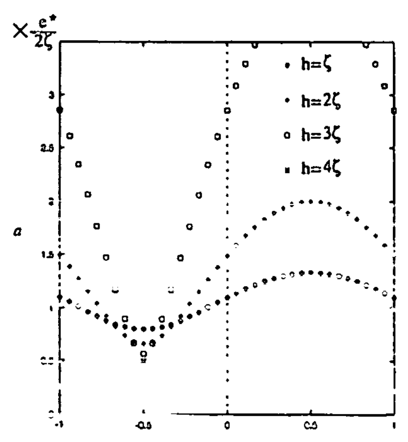


Fig. 2　Amplitude at vibration resonance by parametric excitation ($\nu=2\omega=2\omega_n$).

をとっている．また，同図で縦軸 $a=e^*/2\zeta$ とは線形化したときの共振振幅を示している．この係数励振制御において $a<e^*/2\zeta$ とすることができるときに振動低減に寄与する．$\alpha=-\pi/2$ を選択すると効率がいいことがわかる．また，最もよい選択をした場合でも $a\approx e^*/4\zeta$ までであり，半分の振動低減が限度であることも知れる．

**4・3　$\nu=\omega$ のときの定常状態の振幅**　共振時における励振周波数が加振周波数と等しい場合の定常状態は，式(14)で $\omega=\omega_n$, $du/dt=0$, $dv/dt=0$ とおくことにより得られる．このときの $P_1, Q_1, R_1, S_1, W_u, W_v$ に $\omega=\omega_n$ を代入して得られるものをそれぞれ $P_{1n}$, $Q_{1n}$, $R_{1n}$, $S_{1n}$, $W_{un}$, $W_{vn}$ とおく．$u, v$ は，次のように表すことができる．

$$\left.\begin{aligned} u&=-\frac{S_{1n}W_{un}-Q_{1n}W_{vn}}{P_{1n}S_{1n}-Q_{1n}R_{1n}} \\ v&=-\frac{-R_{1n}W_{un}+P_{1n}W_{vn}}{P_{2n}S_{2n}-Q_{2n}R_{2n}} \end{aligned}\right\} \quad\cdots\cdots\cdots\cdots(21)$$

このときの振幅 $a_p$ は式(8)より計算できる．制御をしない場合の振幅 $a_{p0}$ は，式(17)と同様になる．

**4・4　$\nu=\omega$ のときの振動低減と安定性**　共振時においては，式(14)で $\omega=\omega_n$ として得ることができる．次に $u(t), v(t)$ を求める．また，$u, v$ の初期状態を $u(0)=u_0$, $v(0)=v_0$ とおく．このとき，$h^2=24\zeta/\sqrt{5}$ のときと $h^2\neq 24\zeta/\sqrt{5}$ の場合分けが生じる．また，$u, v$ の初期状態を $u(0)=u_0$, $v(0)=v_0$ とおく．

● $h^2=24\zeta/\sqrt{5}$ のとき

$$\left.\begin{aligned} u(t)&=A_u+B_ut+C_ue^{-2\zeta\omega_nt} \\ v(t)&=A_v+B_vt+C_ve^{-2\zeta\omega_nt} \end{aligned}\right\} \quad\cdots\cdots\cdots\cdots(22)$$

$$A_u=\frac{E_u}{2\zeta\omega_n}-\frac{F_u}{4\zeta^2\omega_n^2},\quad B_u=\frac{F_u}{2\zeta\omega_n}$$

$$C_u=D_u-\frac{E_u}{2\zeta\omega_n}+\frac{F_u}{4\zeta^2\omega_n^2}$$

ただし，

$$D_u=u_0$$

$$E_u=\zeta\omega_nu_0\left(1+\frac{3}{\sqrt{5}}\sin 2\alpha\right)+\frac{\zeta\omega_n}{\sqrt{5}}v_0(2-3\cos 2\alpha)-\frac{\sqrt{6}\zeta g^*}{5^{1/4}\omega_n}\sin\alpha$$

$$F_u=-\frac{\sqrt{6}}{5^{1/4}}\zeta^{3/2}g^*\sin\alpha-\frac{\sqrt{6}}{5^{3/4}}\zeta^{3/2}g^*\cos\alpha-\frac{e^*\zeta\omega_n^2}{2\sqrt{5}}(2-3\cos 2\alpha)$$

$$A_v=\frac{E_v}{2\zeta\omega_n}-\frac{F_v}{4\zeta^2\omega_n^2},\quad B_v=\frac{F_v}{2\zeta\omega_n}$$

$$C_v=D_v-\frac{E_v}{2\zeta\omega_n}+\frac{F_v}{4\zeta^2\omega_n^2}$$

ただし，

$$D_v=v_0$$

$$E_v=-\frac{\zeta\omega_n}{\sqrt{5}}u_0(2+3\cos 2\alpha)+\zeta\omega_nv_0\left(1-\frac{3}{\sqrt{5}}\sin 2\alpha\right)+\frac{\sqrt{6}\zeta g^*}{5^{1/4}\omega_n}\cos\alpha-\frac{e^*\omega_n}{2}$$

$$F_v=\frac{\sqrt{6}}{5^{1/4}}\zeta^{3/2}g^*\cos\alpha-\frac{\sqrt{6}}{5^{3/4}}\zeta^{3/2}g^*\sin\alpha-\frac{e^*\zeta\omega_n^2}{2}\left(1-\frac{3}{\sqrt{5}}\sin 2\alpha\right)$$

振幅 $a$ が発散しないためには，$B_u=B_v=0$ となり，そのときの $h$ は次のようになる．

$$h=-\frac{e^*\omega_n^2}{g^*}\frac{2-3\cos 2\alpha}{\sqrt{5}\sin\alpha+\cos\alpha}\quad\cdots\cdots\cdots\cdots(23)$$

● $h^2\neq 24\zeta/\sqrt{5}$ のとき

$$\left.\begin{aligned} u(t)&=A_u+B_ue^{-pt}+C_ue^{-qt} \\ v(t)&=A_v+B_ve^{-pt}+C_ve^{-qt} \end{aligned}\right\} \quad\cdots\cdots\cdots\cdots(24)$$

$$p=-\omega_n\left(\zeta+\frac{\sqrt{5}}{24}h^2\right),\quad q=-\omega_n\left(\zeta-\frac{\sqrt{5}}{24}h^2\right)$$

$$A_u=\frac{F_u}{pq},\quad B_u=\frac{1}{p-q}\left(D_up-E_u+\frac{F_u}{p}\right)$$

$$C_u=\frac{1}{p-q}\left(-D_uq+E_u-\frac{F_u}{q}\right)$$

ただし，

$$D_u=u_0$$

$$E_u=\zeta\omega_nu_0+\frac{\omega_nh^2v_0}{12}-\frac{hg^*}{2\omega_n}\sin\alpha+\frac{\omega_nh^2}{8}(u_0\sin 2\alpha-v_0\cos 2\alpha)$$

$$F_u=-\frac{h^3g^*}{48}\cos\alpha-\frac{hg^*\zeta}{2}\sin\alpha-\frac{e^*\omega_n^2h^2}{48}(2-3\cos 2\alpha)$$

$$A_v=\frac{F_v}{pq},\quad B_v=\frac{1}{p-q}\left(D_vp-E_v+\frac{F_v}{p}\right)$$

$$C_v=\frac{1}{p-q}\left(-D_vq+E_u-\frac{F_v}{q}\right)$$

ただし，

$$D_v=v_0$$

$$E_v=-\frac{\omega_nh^2u_0}{12}+\zeta\omega_nv_0-\frac{\omega_nh^2}{8}(u_0\cos 2\alpha+v_0\sin 2\alpha)-\frac{e^*\omega_n}{2}+\frac{hg^*}{2\omega_n}\cos\alpha$$

$$F_v=\frac{hg^*\zeta}{2}\cos\alpha-\frac{h^3g^*}{48}\sin\alpha+\frac{e^*\omega_n^2h^2}{16}\sin 2\alpha-\frac{e^*\omega_n^2\zeta}{2}$$

振幅 $a$ が発散しないためには，

Table 1　Meaning of parameters

| $\zeta$ | $\omega$ | $\omega_n$ | $\nu$ | $e^*$ | $Z$ |
|---|---|---|---|---|---|
| 0.05 | 25[Hz] | 25[Hz] | 25[Hz] | 0.1[mm] | 1.28 |

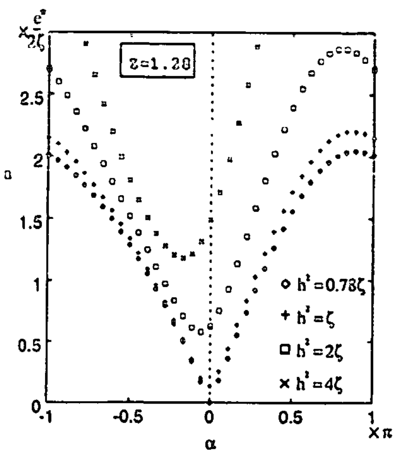


Fig. 3　Amplitude at vibration resonance by parametric excitation ($\nu=\omega=\omega_n$). $Z=1.28$

$$h<\sqrt{\frac{24}{\sqrt{5}}\zeta}\quad i.e.,\ h^2<\frac{24}{\sqrt{5}}\zeta\approx 10.7\zeta \quad \cdots\cdots(25)$$

を満足しなければならない.

ここで, 新しいパラメータ $Z$ を導入する. ここで, $\zeta$ は, $e^*, \omega_n, g^*$ と $Z$ を用いて次のように書くことができる.

$$\zeta=Z\left(\frac{e^*\omega_n^2}{g^*}\right)^2 \quad \cdots\cdots(26)$$

$Z$ は正の値をとり, ベルトに依存する固有の値であり, 大きいほうが減衰のよいベルトとなる. 次に振幅を零にできる $h$ をみつける. ここで, $h^2$ を次のようにおき, これを用いて解析をする.

$$h^2=K\zeta \quad \cdots\cdots(27)$$

ただし, $K$ は正の定数となる.

一方, 振幅が収束値をもつためには, 式(25)の条件より

$$K<\frac{24}{\sqrt{5}} \quad \cdots\cdots(28)$$

となる. $a=0$ とするためには, $F_1^2+F_2^2=0$ が必要になる. $F_1^2+F_2^2$ とは次のように書ける.

$$F_1^2+F_2^2=F_aZ+F_bZ^{1/2}+F_c \quad \cdots\cdots(29)$$

ただし,

$$F_a=\frac{K^3}{2\,304}+\frac{K}{4}$$

$$F_b=-\left(\frac{K^{5/2}}{1\,152}\cos\alpha-\frac{K^{3/2}}{6}\sin\alpha+\frac{K^{1/2}}{2}\cos\alpha\right)$$

$$F_c=\frac{K^2}{2\,304}(13-12\cos 2\alpha)-\frac{K}{16}\sin 2\alpha+\frac{1}{4}$$

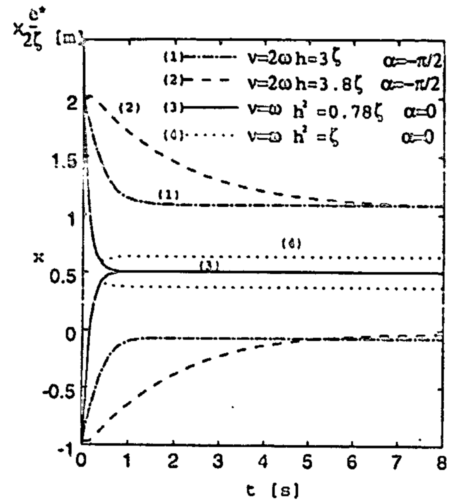


Fig. 4　Amplitude Cuves of belt vibration ($x$)

式(29)は $z^{1/2}$ の二次式とみることができる. 振幅を零とするためには以下の条件が必要になる.

$$\left.\begin{array}{l} Z^{1/2}+\dfrac{F_b}{2F_a}=0 \\ -\dfrac{F_b^2}{4F_a}+F_c=0 \end{array}\right\} \quad \cdots\cdots(30)$$

このとき, 式(30)を満足する $\alpha, K$ の条件は,

$$\left.\begin{array}{l} \alpha=0 \\ KZ=1 \end{array}\right\} \quad \cdots\cdots(31)$$

となる. 式(28)より, 振幅を0にできるベルトの条件は,

$$Z>\frac{\sqrt{5}}{24}\approx 0.093 \quad \cdots\cdots(32)$$

となる. $Z=1.28$ であるベルト系を考え, 加振量 $h^2=\zeta$, $h^2=2\zeta$, $h^2=4\zeta$ とした場合の共振振動振幅を図3に示す. すなわち, この場合には, $a=0$, 約 $h^2=0.78$ で $a=0$ が可能であることがわかる.

## 5. 振動シミュレーション

$h, \alpha$ に対する $x$ の様子を調べるため, 運動方程式(2)に対して, ルンゲクッタ法を用いた数値計算を行う. そのときに用いたパラメータの値を表1に示す. 初期状態は $x=e^*/2\zeta$, $\dot{x}=0.0$ m/s とする.

図4に $\nu=2\omega$, $\nu=\omega$ の場合の時刻に対するベルトの振動 $x$ の振動振幅の包絡線を示す.

● $\nu=2\omega$ では, $a=0$ とすることができない. 約 $a=e^*/4\zeta$ が限界である.

● $Z=1.28$ と減衰の大きいベルトを用いているので, $\nu=\omega$ では, $h, \alpha$ の調整で $a=0$ とすることができる

ことがわかる．また，約1sで収束している．

## 6. おわりに

係数励振によりベルトの共振振動の低減制御を考えたとき，次のように結論をえることができる．

（1） 励振周波数が加振周波数の2倍のとき($\nu=2\omega$)

実際には，$\alpha$の値の誤差などの介入もあることを考えると，$\alpha=-\pi/2$付近で，$h$は$4\zeta$より小さく，かつなるべく$4\zeta$に近い値をとれば，約半分まで振動低減が可能である．

（2） 励振周波数が加振周波数に等しいとき($\nu=\omega$)

一般的には，$\alpha=0$で，$h^2$は$24\zeta/\sqrt{5}$より小さい安定な範囲で，加振すると振動低減になることがわかった．特に，共振時に振幅を零にできるベルトの条件($Z$)が存在することが示された．そのときの完全振動消去の加振条件は，$a=0$，$K=1/Z$であることが示された．

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# VIBRATION AND CONTROL OF AXIALLY MOVING BELT SYSTEM
## (3rd Report: Analysis by Parametric Excitation)

Hiroki OKUBO, Kouetsu TAKANO, Osami MATSUSHITA, Keiji WATANABE, and Yoshi HIRASE

High speed operation is required for industrial machines equipped with belt driving systems. However, a belt has a limited speed due to occurrence of bending mode vibration resonance caused by machine errors such as eccentricity of pulleys. To overcome such resonance of a linear system, a parametric excitation method that provides fluctuation in belt tension is proposed through open-loop vibration control. This paper analyzes two types of fluctuation excitation: one is featured by frequency synchronized with pulley revolution, which is the most effective way, and the other is featured by frequency synchronized with two times the pulley revolution, which is an impractical way. According to analysis of the results of the well-tuned experiments, our control looks promising.

## 1. PREFACE

Belt drives with rubber belts as power transmission means are widely used in various industrial units. Especially, running belt systems take a variety of forms according to the object and usage and are required various performances. High precision and high speed are required in running belts and resonance of lateral oscillation of the belts has thus become a major problem. As research with regard to oscillation restraining control, Chung et al. made theoretical analysis of active oscillation control by boundary displacement input. Also, Rahn et al. made theoretical analysis of oscillation control by parametric excitation. These are based on state feedback control and it is thus considered that there exist practical problems such as installation of sensors and types of actuators causing tension variation.

The present research is directed to restraining lateral oscillation of a flexible means such as a rubber belt. In the previous report, comparative study was made through basic experiments and analysis. In the present report, we are going to make a one-degree-of-freedom theoretical analysis of oscillation restraining of parametric excitation control and to clarify basic concept. We bear in mind a method that is easy to practice, in which torque variation is given to a belt drive motor to cause tension fluctuation of a belt thereby restraining oscillation, as shown in Fig. 1.

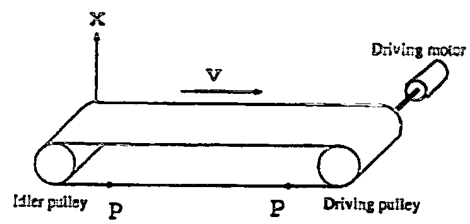


Fig. 1 Driving Belt model

General methods to control oscillation are absorbing oscillating energy through damping force to control oscillation, offsetting vibration waves by generating waves of inverted phase relative to the oscillation that has occurred, and the like. The former method is a way to feedback control the occurrence of oscillation. The latter method is a way to open-loop control to excite so as to cancel oscillation that has occurred. Therefore, both methods differ in object and usage. In this report, we are going to make analysis of flexible or soft model for oscillation control of open-loop using parametric excitation.

## 2. SYSTEM MODEL

As mentioned in the first report, a running belt handled in this research has an extremely small Corioli's force by the speed and has eigenfrequency that will not vary

greatly according to the increase of speed. Therefore, we consider that influence of a speed term does not exist. Also, the source of excitation of the present belt system is an eccentricity of an idler pulley. According to the previous report, equation of motion in this case is as follows:

$$\ddot{x}+\omega_n^2\{1+h\cos(\nu t+\alpha)\}x+2\zeta\omega_n\dot{x}$$
$$=e^*\omega^2\cos\omega t+g^* \quad \cdots\cdots\cdots\cdots\cdots\cdots(1)$$

wherein $x$: displacement, $\omega$ : frequency, $\omega_n$: resonance frequency, h: degree of modulation, $\nu$ : excitation frequency, $\alpha$ : phase, $t$: time, $e$: eccentricity, $g$: gravitational acceleration, $e^*=(2/\pi)e$, $g^*=(4/\pi)g$. $\zeta$, $e^*$, $h$: extremely small value, and $\omega \fallingdotseq \omega_n$ .

The second term on the left side designates tension variation and the third term damping. The first term on the right side designates eccentric centrifugal force. A number of reports were published regarding stability of March equation in the case of small non-linearity.

The gist of this report is to consider whether resonance oscillation control is possible with the parameters of ($h$) and ( $\alpha$ ) for tension variation. Non-linear oscillation analysis is made using asymptotic expansion method. In order to make a later analysis easy, the differential equation is rewritten as follows:

$$\ddot{x}+\omega^2 x=(\omega^2-\omega_n^2)x-2\zeta\omega_n\dot{x}+e^*\omega^2\cos\omega t$$
$$-\omega_n^2 h\cos(\nu t+\alpha)x+g^* \quad \cdots\cdots\cdots\cdots\cdots\cdots (2)$$

## 3. OSCILLATION ANALYSIS FOR FLEXIBLE SYSTEM

### *3.1 Arrangement of Differential Equation*

The following gives an analogous solution to equation (2).

$$x=a\cos(\omega t+\vartheta)+\varepsilon u_1(a,\vartheta,\nu t) \quad \cdots\cdots\cdots\cdots (3)$$

wherein $\alpha$ and $\theta$ designate amplitude and phase, respectively, which are not constants but variables that vary slowly due to non-linearity. They are supposed to be as follows:

$$\frac{da}{dt}=\varepsilon A_1(a,\vartheta)+\varepsilon^2 A_2(a,\vartheta)$$
$$\frac{d\vartheta}{dt}=\varepsilon B_1(a,\vartheta)+\varepsilon^2 B_2(a,\vartheta)$$

The left side of equation (2) is rewritten as follows:

$$
\begin{aligned}
\ddot{x}+\omega^2 x = {} & \varepsilon\Big(-2A_1\omega\sin\psi \\
& -2aB_1\omega\cos\psi+\frac{\partial^2 u_1}{\partial t^2}+\omega^2 u_1\Big) \\
& +\varepsilon^2\Big[\Big(A_1\frac{\partial A_1}{\partial a}+B_1\frac{\partial A_1}{\partial\vartheta}-aB_1^2-2a\omega B_2\Big)\cos\psi \\
& +\Big\{-2A_1B_1-2\omega A_2-a\Big(A_1\frac{\partial B_1}{\partial a} \\
& +B_1\frac{\partial B_1}{\partial\vartheta}\Big)\Big\}\sin\psi+2A_1\frac{\partial^2 u_1}{\partial a\partial t}+2B_1\frac{\partial^2 u_1}{\partial\vartheta\partial t}\Big] \qquad \cdots\cdots\cdots\cdots(4)
\end{aligned}
$$

In this paper, we consider the case of $v=\omega$ where excitation is carried out at the same frequency as resonance frequency and the case of $v=2\omega$ where excitation is carried out at the doubled frequency. When ignoring the small second order term and arranging the right side of equation (2) to obtain the following: In this case, $\psi=\omega t+\theta$.

$$
\begin{aligned}
& (\omega^2-\omega_n^2)x=(\omega^2-\omega_n^2)a\cos\psi \\
& -2\zeta\omega_n\dot{x}=2\zeta\omega_n a\omega\sin\psi \\
& \quad +\varepsilon\Big(A_1\cos\psi-aB_1\sin\psi+\frac{\partial u_1}{\partial t}\Big) \\
& e^*\omega^2\cos\omega t=e^*\omega^2(\cos\vartheta\cos\psi+\sin\vartheta\sin\psi)
\end{aligned}
$$

At $\nu = \omega$,

$$-\omega_n^2 h \cos(\nu t + \alpha) x$$
$$= -\frac{\omega_n^2 h a}{2} \{\cos(2\phi + \alpha - \vartheta)$$
$$+ \cos(\alpha - \vartheta)\}$$
$$- \omega^2 h \varepsilon u_1 \cos(\omega t + \alpha)$$

At $\nu = 2\omega$,

$$-\omega_n^2 h \cos(\nu t + \alpha) x$$
$$= -\frac{\omega_n^2 h a}{2} \{\cos(\alpha - 2\vartheta) \cos \phi$$
$$+ \cos(\alpha - 2\vartheta) \cos 3\phi$$
$$- \sin(\alpha - 2\vartheta) \sin 3\phi$$
$$- \sin(\alpha - 2\vartheta) \sin \phi\}$$

Both sides of equation (2) are made equal to each other using the above-mentioned equations.

### *3.2 Analysis of Amplitude and Phase (at $\nu = 2\omega$)*

In the case where the excitation frequency is twice as high as the frequency, or $\nu = 2\omega$, by comparing $\cos \phi$ with $\sin \phi$ in the equation that was acquired by ignoring the terms more than $\varepsilon^2$, the following equations can be obtained:

$$
\left.
\begin{aligned}
\frac{da}{dt} &= -\zeta\omega_n a - \frac{e^*\omega}{2}\sin\vartheta \\
&\quad -\frac{\omega_n^2 h}{4\omega} a \sin(\alpha - 2\vartheta) \\
\frac{d\vartheta}{dt} &= -\frac{\omega^2-\omega_n^2}{2\omega} - \frac{\varepsilon^*\omega}{2a}\cos\vartheta \\
&\quad +\frac{\omega_n^2}{4\omega}\cos(\alpha-2\vartheta)
\end{aligned}
\right\}
\cdots\cdots\cdots\cdots\cdots (5)
$$

$u_1$ will be obtained from direct current and harmonic component that are not coefficient in the basic wave of cos $\psi$ and sin $\psi$.

$$
\varepsilon\left(\frac{\partial^2 u_1}{\partial t^2} + \omega^2 u_1\right) = g^* - \frac{\omega_n^2 h a}{2}\{\cos(\alpha-2\vartheta)\cos 3\psi - \sin(\alpha-2\vartheta)\sin 3\psi\} \cdots\cdots\cdots\cdots\cdots\cdots (6)
$$

Therefore, $\varepsilon u_1$ will be given as a special solution to the above equation.

$$
\varepsilon u_1 = \frac{g^*}{\omega^2} + C_1\cos 3\psi + C_2\sin 3\psi \cdots\cdots\cdots\cdots (7)
$$

$$
C_1 = \frac{\omega_n^2 h a}{16\omega^2}\cos(\alpha-2\vartheta)
$$

$$
C_2 = -\frac{\omega_n^2 h a}{16\omega^2}\sin(\alpha-2\vartheta)
$$

Here, new variable $u(t)= a\cos\theta$, $v(t)= a\sin\theta$ are introduced. Then, $a$ and $\theta$ are rewritten into the following:

$$\left.\begin{aligned} a^2 &= u^2+v^2 \\ \vartheta &= arc\cdot\tan\frac{v}{u} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (8)$$

Using these equations, equation (5) will be rewritten as follows:

$$\frac{d}{dt}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} P_2 & Q_2 \\ R_2 & S_2 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{e^*\omega}{2} \end{bmatrix} \quad \cdots\cdots\cdots (9)$$

Wherein,

$$P_2 = -\zeta\omega_n - \frac{\omega_n^2 h}{4\omega}\sin\alpha$$

$$Q_2 = \frac{\omega^2-\omega_n^2}{2\omega} + \frac{\omega_n^2 h}{4\omega}\cos\alpha$$

$$R_2 = -\frac{\omega^2-\omega_n^2}{2\omega} + \frac{\omega_n^2 h}{4\omega}\cos\alpha$$

$$S_2 = -\zeta\omega_n + \frac{\omega_n^2 h}{4\omega}\sin\alpha$$

*3.3 Analysis of Amplitude and Phase (at $\nu=\omega$)*

In the case where the excitation frequency equals to the frequency, or $\nu=\omega$, by comparing coefficients of basic

waves of *cos* $\phi$ and *sin* $\phi$ with each other in the equation that was acquired by ignoring the terms more than $\varepsilon^2$, the following equations can be obtained:

$$\left.\begin{aligned}\frac{da}{dt}&=-\zeta\omega_n a-\frac{e^*\omega}{2}\sin\vartheta\\ \frac{d\vartheta}{dt}&=-\frac{\omega^2-\omega_n^2}{2\omega}-\frac{e^*\omega}{2a}\cos\vartheta\end{aligned}\right\}\cdots\cdots\cdots\cdots\cdots(10)$$

$\varepsilon\, u_1$ will be obtained as a special solution to non-basic waves.

$$\varepsilon\left(\frac{\partial^2 u_1}{\partial t^2}+\omega^2 u_1\right)=g^*-\frac{\omega_n^2 ha}{2}\{\cos(2\psi+\alpha-\vartheta)+\cos(\alpha-\vartheta)\}\cdots\cdots\cdots\cdots\cdots(11)$$

Therefore, $\varepsilon\, u_1$ will be given as follows:

$$\varepsilon u_1=\frac{g^*}{\omega^2}-\frac{\omega_n^2 ha}{2\omega^2}\cos(\alpha-\vartheta)+\frac{\omega_n^2 ha}{6\omega^2}\cos(2\psi+\alpha-\vartheta)\cdots\cdots\cdots\cdots\cdots(12)$$

When the excitation frequency is equal to the frequency, a term of modulation degree $h$ is not included in the analysis up to a minute term of degree one, as can be seen in equation (10). Therefore, effect of parametric

excitation cannot be found. Consequently, analysis including minute terms of up to degree two will be carried out. The minute term of degree two at the first term on the right side of equation (2) will be as follows:

$$\varepsilon(\omega^2-\omega_n^2)u_1=(\omega^2-\omega_n^2)\left\{\frac{g^*}{\omega^2}-\frac{\omega_n^2 ha}{2\omega^2}\cos(\alpha-\vartheta)\right.$$
$$\left.+\frac{\omega_n^2 ha}{6\omega^2}\cos(2\psi+\alpha-\vartheta)\right\}$$

The minute term of degree two at the second term on the right side of equation (2) will be as follows:

$$-2\zeta\omega_n\left(\frac{da}{dt}\cos\psi-a\frac{d\vartheta}{dt}\sin\psi+\varepsilon\frac{\partial u_1}{\partial t}\right)$$

The minute term of degree two does not exist at the third term on the right side of equation (2). The minute term of degree two at the fourth term on the right side of equation (2) will be as follows:

$$-\omega_n^2 h\cos(\omega t+\alpha)\varepsilon u_1$$
$$=-\left(\frac{\omega_n}{\omega}\right)^2 h\left\{g^*-\frac{\omega^2 ha}{2}\cos(\alpha-\vartheta)\right\}$$
$$\times\cos(\alpha-\vartheta)\cos\psi$$
$$+\left(\frac{\omega_n}{\omega}\right)^2 h\left\{g^*-\frac{\omega^2 ha}{2}\cos(\alpha-\vartheta)\right\}$$
$$\times\sin(\alpha-\vartheta)\sin\psi$$
$$-\left(\frac{\omega_n^4}{12\omega}\right)h^2 a\{\cos\psi-\cos(3\psi+2\alpha-2\vartheta)\}$$

$da/dt$, $d\theta/dt$ will be obtained as follows:

$$\left.\begin{aligned}
\frac{da}{dt} &= -\zeta\omega_n a - \frac{e^*\omega}{2}\sin\vartheta \\
&\quad + \frac{\omega_n^4 h^2 a}{8\omega^3}\sin 2(a-\vartheta) - \frac{\omega_n^2 h g^*}{2\omega^3}\sin(a-\vartheta) \\
\frac{d\vartheta}{dt} &= -\frac{\omega^2-\omega_n^2}{2\omega} - \frac{(\omega^2-\omega_n^2)^2}{8\omega^3} \\
&\quad - \frac{\omega_n^4 h^2}{12\omega^3} - \frac{e^*\omega}{2a}\cos\vartheta \\
&\quad - \frac{\omega_n^4 h^2}{8\omega^3}\cos 2(a-\vartheta) + \frac{\omega_n^2 h g^*}{2a\omega^3}\cos(a-\vartheta)
\end{aligned}\right\} \qquad \cdots\cdots\cdots\cdots (13)$$

Using $u(t)$ and $v(t)$, equation (13) is rewritten into the following:

$$\frac{d}{dt}\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} P_1 & Q_1 \\ R_1 & S_1 \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} W_u \\ W_v \end{bmatrix} \qquad \cdots\cdots\cdots\cdots\cdots (14)$$

Wherein,

$$\begin{aligned}
P_1 &= -\zeta\omega_n + \frac{\omega_n^4 h^2}{8\omega^3}\sin 2a \\
Q_1 &= \frac{\omega^2-\omega_n^2}{2\omega} + \frac{(\omega^2-\omega_n^2)^2}{8\omega^3} + \frac{\omega_n^4 h^2}{12\omega^3} - \frac{\omega_n^4 h^2}{8\omega^3}\cos 2a \\
R_1 &= -\frac{\omega^2-\omega_n^2}{2\omega} - \frac{(\omega^2-\omega_n^2)^2}{8\omega^3} \\
&\quad - \frac{\omega_n^4 h^2}{12\omega^3} - \frac{\omega_n^4 h^2}{8\omega^3}\cos 2a \\
S_1 &= -\zeta\omega_n - \frac{\omega_n^4 h^2}{8\omega^3}\sin 2a \\
W_u &= -\frac{\omega_n^2 h g^*}{2\omega^3}\sin a, \quad W_v = \frac{\omega_n^2 h g^*}{2\omega^3}\cos a - \frac{e^*\omega}{2}
\end{aligned}$$

## 4. OSCILLATION ANALYSIS AT RESONANCE

In this paper, we are going to proceed with oscillation analysis at the time of resonance. At resonance, $\omega = \omega_n$ is satisfied. Then, we are going to obtain resonance oscillation at steady state in the case of $\nu = \omega$ and $\nu = 2\omega$ of parametric excitation and discuss stability of the steady state. Finally, we seek the condition in which reduction in oscillation is stably possible by parametric oscillation.

### *4.1 Amplitude at Steady State (at $\nu = 2\omega$)*

Steady state where excitation frequency is twice greater than the frequency at resonance can be obtained by substituting $\omega_n$ for $\omega$, $0$ for $du/dt$, and $0$ for $dv/dt$ in equation (9). Then, $P_{2n}$, $Q_{2n}$, $R_{2n}$, $S_{2n}$ are substituted for $P_2$, $Q_2$, $R_2$, $S_2$, respectively. $u$ and $v$ are shown as follows:

$$\left.\begin{aligned} u &= -\frac{Q_{2n}}{P_{2n}S_{2n} - Q_{2n}R_{2n}} \frac{e^* \omega_n}{2} \\ v &= \frac{P_{2n}}{P_{2n}S_{2n} - Q_{2n}R_{2n}} \frac{e^* \omega_n}{2} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots (15)$$

As the amplitude is shown by $a_p$,

$$a_p = \frac{2e}{|16\zeta^2 - h^2|} \sqrt{h^2 + 8\zeta h \sin \alpha + 16\zeta^2} \quad \cdots\cdots (16)$$

is obtained. Without control, $h$ equals to $0$.

Amplitude $a_{p0}$ at resonance without control is shown by

$$a_{p0}=\frac{e^*}{2\zeta} \cdots\cdots (17)$$

That stands for amplitude for resonance oscillation in linear system, which is obtained by multiplying eccentricity by resonance magnification.

*4.2 Reduction in Oscillation and Stability at $\nu=2\omega$*

At the time of resonance, $\omega_n$ is substituted for $\omega$ in equation (9). There are two case of $h=4\zeta$ and $h\neq 4\zeta$. Also, initial state of $u$ and $v$ is set at $u(0)=u_0$, $v(0)=v_0$, respectively.

●*At $h=4\zeta$*

$$\left.\begin{aligned} u(t)&=A_u+B_ut+C_ue^{-2\zeta\omega_n t} \\ v(t)&=A_v+B_vt+C_ve^{-2\zeta\omega_n t} \end{aligned}\right\} \cdots\cdots (18)$$

wherein,

$$A_u=\frac{e^*\cos\alpha}{8\zeta}+\frac{1}{2}\{u_0(1-\sin\alpha)+v_0\cos\alpha\}$$

$$B_u=-\frac{1}{4}e^*\omega_n\cos\alpha$$

$$C_u=-\frac{e^*\cos\alpha}{8\zeta}+\frac{1}{2}\{u_0(1+\sin\alpha)-v_0\cos\alpha\}$$

$$A_v=-\frac{e^*}{8\zeta}(1-\sin\alpha)$$

$$+\frac{1}{2}\{u_0\cos\alpha+v_0(1+\sin\alpha)\}$$

$$B_v = -\frac{1}{4} e^* \omega_n (1 + \sin \alpha)$$

$$C_v = -\frac{e^*}{8\zeta}(1 - \sin \alpha) - \frac{1}{2}\{u_0 \cos \alpha - v_0(1 - \sin \alpha)\}$$

In order that amplitude $a$ does not diverge, equation, $B_u = B_v = 0$ should be satisfied. At this time, $a$ should satisfy the following condition:

$$\alpha = -\frac{\pi}{2} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (19)$$

In this case, $a$ does not converge if $a$ is even slightly out of the condition of equation (19). However, in actual machines, it is difficult to set $a$ without any error. Therefore, $h = 4\zeta$ is not a practical case.

●At $h \neq 4\zeta$

$$
\left.
\begin{aligned}
u(t) &= A_u + B_u e^{-\omega_n(\zeta+h/4)t} + C_u e^{-\omega_n(\zeta-h/4)t} \\
v(t) &= A_v + B_v e^{-\omega_n(\zeta+h/4)t} + C_v e^{-\omega_n(\zeta-h/4)t}
\end{aligned}
\right\} \qquad \cdots\cdots\cdots\cdots (20)
$$

wherein,

$$A_u = \frac{2e^* h \cos \alpha}{h^2 - 16\zeta^2}$$

$$B_u = -\frac{e^* \cos \alpha}{h + 4\zeta} + \frac{u_0}{2}(1 + \sin \alpha) - \frac{v_0}{2} \cos \alpha$$

$$C_u = -\frac{e^* \cos \alpha}{h - 4\zeta} + \frac{u_0}{2}(1 - \sin \alpha) + \frac{v_0}{2} \cos \alpha$$

$$A_v = \frac{2e^*(4\zeta + h \sin \alpha)}{h^2 - 16\zeta^2}$$

$$B_v = -\frac{e^*(1 - \sin \alpha)}{h + 4\zeta} - \frac{u_0}{2} \cos \alpha + \frac{v_0}{2}(1 - \sin \alpha)$$

$$C_v = -\frac{e^*(1 + \sin \alpha)}{h - 4\zeta} + \frac{u_0}{2} \cos \alpha + \frac{v_0}{2}(1 + \sin \alpha)$$

In order that amplitude $a$ does not diverge, $h < 4\zeta$ should be satisfied so that the third term of equation (20) can damp.

Next, Fig. 2 shows amplitude $a$ at the time of convergence. Horizontal axis of Fig. 2 designates phase angle

$\alpha$. In addition, amplitude at $h=4\zeta$ is the one when the initial value is set at $u_0=0$ and $v_0=0$. Also, in Fig. 2, vertical axis $a=e^*/2\zeta$ designates resonance amplitude at the time of linearization. In this parametric excitation control, when $a<e^*/2\zeta$ can be achieved it contributes the reduction in oscillation. It can be found that when $\alpha=-\pi/2$ is selected efficiency becomes better. Also, even in the case where best selection was made, $a$ is up to $a \fallingdotseq e^*/4\zeta$. It can be seen that reduction in oscillation is limited to a half of the oscillation.

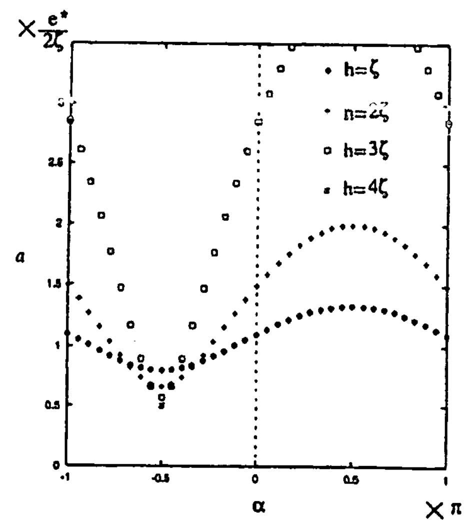


Fig. 2 Amplitude at vibration resonance by parametric excitation ($\nu=2\omega=2\omega_n$).

### 4.3 Amplitude at Steady State at $\nu = \omega$

Steady state where excitation frequency at resonance is equal to the frequency can be obtained by substituting $\omega_n$ for $\omega$, 0 for $du/dt$, and 0 for $dv/dt$ in equation (14). Then, $P_{1n}$, $Q_{1n}$, $R_{1n}$, $S_{1n}$, $W_{un}$, and $W_{vn}$ are substituted for $P_1$, $Q_1$, $R_1$, $S_1$, $W_u$, and $W_v$, respectively. $u$ and $v$ are shown as follows:

$$\left.\begin{aligned} u &= -\frac{S_{1n}W_{un} - Q_{1n}W_{vn}}{P_{1n}S_{1n} - Q_{1n}R_{1n}} \\ v &= -\frac{-R_{1n}W_{un} + P_{1n}W_{vn}}{P_{2n}S_{2n} - Q_{2n}R_{2n}} \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots (21)$$

Amplitude $a_p$ can be calculated by equation (8). Amplitude $a_{po}$ without control is similar to equation (17).

### 4.4 Reduction in Oscillation and Stability at $\nu = \omega$

At the time of resonance, $\omega_n$ is substituted for $\omega$ in equation (14). Then, $u(t)$ and $v(t)$ will be obtained. Also, initial state of $u$ and $v$ is set at $u(0)=u_0$, $v(0)=v_0$, respectively. There are two case of $h^2=24\zeta/\sqrt{5}$ and $h^2 \neq 24\zeta/\sqrt{5}$.

● *At* $h^2=24\zeta/\sqrt{5}$

$$\left.\begin{aligned} u(t) &= A_u + B_u t + C_u e^{-2\zeta\omega_n t} \\ v(t) &= A_v + B_v t + C_v e^{-2\zeta\omega_n t} \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots (22)$$

$$A_u = \frac{E_u}{2\zeta\omega_n} - \frac{F_u}{4\zeta^2\omega_n^2},\ B_u = \frac{F_u}{2\zeta\omega_n}$$

$$C_u = D_u - \frac{E_u}{2\zeta\omega_n} + \frac{F_u}{4\zeta^2\omega_n^2}$$

Wherein,

$$D_u = u_0$$

$$E_u = \zeta\omega_n u_0\left(1+\frac{3}{\sqrt{5}}\sin 2\alpha\right) + \frac{\zeta\omega_n}{\sqrt{5}}v_0(2-3\cos 2\alpha) - \frac{\sqrt{6}\zeta g^*}{5^{1/4}\omega_n}\sin\alpha$$

$$F_u = -\frac{\sqrt{6}}{5^{1/4}}\zeta^{3/2}g^*\sin\alpha - \frac{\sqrt{6}}{5^{3/4}}\zeta^{3/2}g^*\cos\alpha - \frac{e^*\zeta\omega_n^2}{2\sqrt{5}}(2-3\cos 2\alpha)$$

$$A_v = \frac{E_v}{2\zeta\omega_n} - \frac{F_v}{4\zeta^2\omega_n^2},\quad B_v = \frac{F_v}{2\zeta\omega_n}$$

$$C_v = D_v - \frac{E_v}{2\zeta\omega_n} + \frac{F_v}{4\zeta^2\omega_n^2}$$

Wherein,

$$D_v = v_0$$

$$E_v = -\frac{\zeta\omega_n}{\sqrt{5}} u_0 (2+3\cos 2\alpha) + \zeta\omega_n v_0 \left(1-\frac{3}{\sqrt{5}}\sin 2\alpha\right) + \frac{\sqrt{6}\zeta g^*}{5^{1/4}\omega_n}\cos\alpha - \frac{e^*\omega_n}{2}$$

$$F_v = \frac{\sqrt{6}}{5^{1/4}}\zeta^{3/2} g^* \cos\alpha - \frac{\sqrt{6}}{5^{3/4}}\zeta^{3/2} g^* \sin\alpha - \frac{e^*\zeta\omega_n^2}{2}\left(1-\frac{3}{\sqrt{5}}\sin 2\alpha\right)$$

In order that amplitude $a$ does not diverge, $B_u = B_v = 0$ should be satisfied. Then, $h$ will be as follows:

$$h = -\frac{e^*\omega_n^2}{g^*}\frac{2-3\cos 2\alpha}{\sqrt{5}\sin\alpha+\cos\alpha} \quad \cdots\cdots\cdots\cdots\cdots (23)$$

●At $h^2 \neq 24\zeta/\sqrt{5}$

$$\left.\begin{aligned} u(t) &= A_u + B_u e^{-pt} + C_u e^{-qt} \\ v(t) &= A_v + B_v e^{-pt} + C_v e^{-qt} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots\cdots (24)$$

$$p = -\omega_n\left(\zeta + \frac{\sqrt{5}}{24}h^2\right),\quad q = -\omega_n\left(\zeta - \frac{\sqrt{5}}{24}h^2\right)$$

$$A_u = \frac{F_u}{pq},\quad B_u = \frac{1}{p-q}\left(D_u p - E_u + \frac{F_u}{p}\right)$$

$$C_u = \frac{1}{p-q}\left(-D_u q + E_u - \frac{F_u}{q}\right)$$

Wherein,

$$D_u = u_0$$

$$E_u = \zeta\omega_n u_0 + \frac{\omega_n h^2 v_0}{12} - \frac{hg^*}{2\omega_n}\sin\alpha + \frac{\omega_n h^2}{8}(u_0 \sin 2\alpha - v_0 \cos 2\alpha)$$

$$F_u = -\frac{h^3 g^*}{48}\cos\alpha - \frac{hg^*\zeta}{2}\sin\alpha - \frac{e^*\omega_n^2 h^2}{48}(2 - 3\cos 2\alpha)$$

$$A_v = \frac{F_v}{pq},\quad B_v = \frac{1}{p-q}\left(D_v p - E_v + \frac{F_v}{p}\right)$$

$$C_v = \frac{1}{p-q}\left(-D_v q + E_u - \frac{F_v}{q}\right)$$

Wherein,

$$D_v = v_0$$

$$E_v = -\frac{\omega_n h^2 u_0}{12} + \zeta\omega_n v_0 - \frac{\omega_n h^2}{8}(u_0 \cos 2\alpha + v_0 \sin 2\alpha) - \frac{e^*\omega_n}{2} + \frac{hg^*}{2\omega_n}\cos\alpha$$

$$F_v = \frac{hg^*\zeta}{2}\cos\alpha - \frac{h^3 g^*}{48}\sin\alpha + \frac{e^*\omega_n^2 h^2}{16}\sin 2\alpha - \frac{e^*\omega_n^2\zeta}{2}$$

In order for amplitude $a$ not to diverge,

$$h < \sqrt{\frac{24}{\sqrt{5}}\zeta} \text{ i.e., } h^2 < \frac{24}{\sqrt{5}}\zeta \approx 10.7\zeta \quad \cdots\cdots (25)$$

should be satisfied.

Here, a new parameter is introduced. $\zeta$ can be rewritten into the following using $e^*$, $\omega_n$, $g^*$, and $Z$:

$$\zeta = Z\left(\frac{e^* \omega_n^2}{g^*}\right)^2 \quad \cdots\cdots (26)$$

$Z$ takes a positive value, which is specific to a belt. As the value becomes greater, the better damping properties the belt has. Next, we are going to find $h$ that makes amplitude zero. Here, by substituting $K\zeta$ for $h^2$, analysis will be carried out. That is,

$$h^2 = K\zeta \quad \cdots\cdots (27)$$

wherein, $K$ is a positive constant.

On the other hand, in order for amplitude to have a convergent value, from the condition of equation (25),

$$K < \frac{24}{\sqrt{5}} \quad \cdots\cdots (28)$$

$F_u^2 + F_v^2 = 0$ is required to achieve $a=0$. $F_u^2 + F_v^2$ will be rewritten into the following:

$$F_u^2 + F_v^2 = F_a Z + F_b Z^{1/2} + F_c \quad \cdots\cdots (29)$$

wherein,

$$F_a=\frac{K^3}{2\,304}+\frac{K}{4}$$
$$F_b=-\left(\frac{K^{5/2}}{1\,152}\cos\alpha-\frac{K^{3/2}}{8}\sin\alpha+\frac{K^{1/2}}{2}\cos\alpha\right)$$
$$F_c=\frac{K^2}{2\,304}(13-12\cos 2\alpha)-\frac{K}{16}\sin 2\alpha+\frac{1}{4}$$

Equation (29) can be considered a quadratic expression of $z^{1/2}$. In order to make amplitude zero, the following condition is required:

$$\left.\begin{aligned} Z^{1/2}+\frac{F_b}{2F_a}&=0\\ -\frac{F_b^2}{4F_a}+F_c&=0\end{aligned}\right\} \quad\cdots\cdots(30)$$

At this time, the condition of $a$ and $K$ that satisfies equation (30) is as follows:

$$\left.\begin{aligned} \alpha&=0\\ KZ&=1\end{aligned}\right\} \quad\cdots\cdots(31)$$

From equation (28), the following is a belt condition that can make amplitude zero:

$$Z>\frac{\sqrt{5}}{24}\approx 0.093 \quad\cdots\cdots(32)$$

Here, we assume a belt system of $Z$=1.28. Fig. 3 shows an amplitude at resonance oscillation in the case of $h^2=\zeta$, $h^2=2\zeta$, $h^2=4\zeta$. That is, in this case, it can be seen that $a$=0 is possible at $\alpha$=0 and $h^2 \fallingdotseq 0.78$.

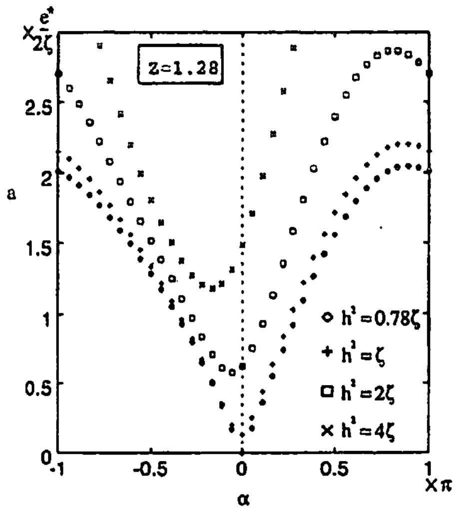


Fig. 3 Amplitude at vibration resonance by parametric excitation ($\nu=\omega=\omega_n$). $Z$=1.28

## 5. OSCILLATION SIMULATION

Numerical calculation was carried out in equation of motion (2) in order to search variation of $x$ relative to $h$ and $a$. Parameters used here are shown in Table 1. Initial state is $x=e^*/\zeta$ and $\dot{x}$=0.0 m/s.

Table 1 Meaning of parameters

| $\zeta$ | $\omega$ | $\omega_n$ | $\nu$ | $e^*$ | $Z$ |
|---|---|---|---|---|---|
| 0.05 | 25[$Hz$] | 25[$Hz$] | 25[$Hz$] | 0.1[$mm$] | 1.28 |

Fig. 4 shows envelopes of oscillation amplitudes of belt oscillation $x$ relative to time in the case of $\nu = 2\omega$ and $\nu = \omega$.

- At $\nu = 2\omega$, $a$=0 cannot be obtained。 $a \doteqdot e^*/4\zeta$ is the limit.
- Since a belt with a higher damping properties of $Z$=1.28 is used, at $\nu = \omega$, it is found that $a$=0 can be obtained with adjustment of $h$ and $a$. Also, it converges in about 1s.

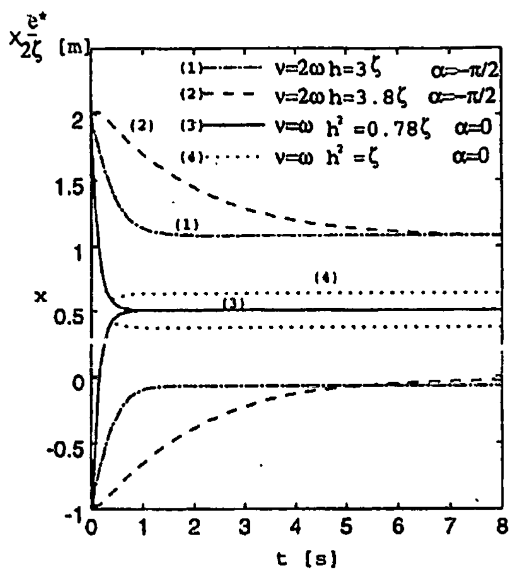


Fig. 4 Amplitude Cuves of belt vibration ($x$)

## 6. CONCLUSION

When we consider reduction control of resonance oscillation of a belt via parametric excitation, we can conclude as follows:

(1) *When excitation frequency is twice greater than the frequency or $\nu = 2\omega$;*

In fact, since errors of $a$ could be introduced, if we take a value of $h$ that is smaller than $4\zeta$ and that is as close to $4\zeta$ as possible around $\alpha=-\pi/2$, reduction in oscillation can be achieved up to approximately half of it.

(2) *When excitation frequency is equal to the frequency or* $\nu=\omega$;

Generally, it has been found that oscillation can be reduced when excited at $\alpha=0$ and in the stable range of $h^2<24\zeta/\sqrt{5}$ . Especially, existence of belt condition ($Z$) was shown that can make amplitude zero at the time of resonance. It was also shown that excitation condition of perfect cancellation of oscillation is $\alpha=0$ and $K=1/Z$.